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A stochastic model for particle convective heat transfer in gas–solid fluidized beds

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Abstract—Particle convection is the predominant mode of heat transfer in gas–solid fluidized beds of small particles at temperatures below 650 K and near-ambient pressures. The mechanism of heat transfer is unsteady-state conduction between the heat transfer surface and the hot bed particles which contact it in a recurrent, aperiodic manner. The particle–surface contacting is treated here as a stochastic process with the inter-packet periods being described by a constant-rate Poisson distribution and the contact periods by a negative exponential distribution. These features of the contacting process are incorporated into the packet heat transfer model of Baskakov, and expressions for the heat transfer coefficient and its variance are derived. The contact period distribution data of Ozkaynak and Chen for a bed of 245 μm glass beads at three fluidizing velocities is employed to predict the heat transfer coefficient and its variance at the three velocities. The predicted heat transfer coefficients are found to be in excellent agreement with the measured heat transfer coefficients. However, paucity of simultaneous contact-period distribution and heat transfer data has limited further verification of this model.

INTRODUCTION

HEAT TRANSFER rates to surfaces immersed in gas fluidized beds are several times those to surfaces in flowing gas streams. The enhancement in heat transfer occurs due to the contact between the hot, solid fluidized particles and the heat transfer surface. This mode of heat transfer, termed as particle convection, supplements the gas convective and radiative modes of heat transfer in gas fluidized beds, and is the dominant mode in small particle fluidized beds. Radiative heat transfer constitutes only 15% of the total heat transfer at a bed temperature of 1175 K, and is negligible at temperatures below 675 K [1]. Gas convective heat transfer is smaller than particle convective heat transfer by a couple of orders of magnitude in small particle beds at near-atmospheric pressures [2]. It has been argued on hydrodynamic principles that gas convective heat transfer is negligible for beds with $Re_{mf} < 10$, or $Ar < 21\,700$ [3].

The particle convective heat transfer process has been extensively studied, and several models have been proposed. These models have been discussed and evaluated in various monographs and reviews [4–7], and the approach which is now accepted is broadly envisioned to occur as follows: hot, solid particles move from the bulk of the bed and contact the surface in a recurrent, aperiodic manner. The mode of heat transfer during the contact period is unsteady-state heat conduction. The earliest heat transfer model is due to Mickley and Fairbanks [8], who conceptualized the volume of the bed emulsion that is affected by the thermal process during a contact period as a 'packet', and related the instantaneous particle convective heat transfer coefficient, $h_p(t)$ (or its inverse, the packet resistance, R_p), to the packet properties as

$$h_p(t) = 1/R_p = [k_e C_s \rho_e / t_c \pi]^{1/2}. \qquad (1)$$

Equation (1) overpredicts h_p, especially for short packet contact periods, and implies that it tends to infinity as t_c tends to zero. Baskakov [9] argued that the packet continuum is significantly disturbed close to the heat transfer surface and suggested that an additional 'contact' resistance, R_c, occurs next to the heat transfer surface, in series with R_p. The instantaneous packet heat transfer coefficient, $h_p(t)$, for the constant heat transfer case is then given by [6]

$$h_p(t) = \mathrm{erfc}\,(X) \exp\,(X^2)/R_c \qquad (2)$$

where

$$X = R_p/R_c\sqrt{\pi}. \qquad (3)$$

Xavier and Davidson [10] suggest the contact resistance can be expressed as

$$R_c = d_p/k_g b. \qquad (4)$$

The assumption in describing the contact resistance by equation (4) is that the contact resistance consists of a gas layer of average thickness, d/b, in between the heat transfer surface and the emulsion. However, the presence of such a film is doubtful, and the contact resistance is best explained in terms of the real contact geometry between particles and the surface. Decker and Glicksman [11] point out that actual particle-to-surface contact occurs only at a limited number of microscopic roughness elements. Their model of the total conduction heat transfer within the particle in

NOMENCLATURE

A	dimensionless variable, Y^2/C
$\bar{A}$	value of A at the expected value of C
Ar	Archimedes number, $gd_p^3\rho_g(\rho_s-\rho_g)/\mu^2$
b	constant
C	inverse of mean packet contact period [s^{-1}]
$\bar{C}$	expected value of C [s^{-1}]
C_s	heat capacity of solid particles [$J\ kg^{-1}\ K^{-1}$]
d_p	bed particle diameter [m]
D	average particle arrival rate [s^{-1}]
F	survivor distribution function
g	acceleration due to gravity [$m\ s^{-2}$]
h_p	instantaneous packet to surface heat transfer coefficient [$W\ m^{-2}\ K^{-1}$]
$\bar{h}_p$	average particle convective heat transfer coefficient [$W\ m^{-2}\ K^{-1}$]
$\bar{h}_{p,e}$	expected value of the average particle convective heat transfer coefficient [$W\ m^{-2}\ K^{-1}$]
k_e	thermal conductivity of emulsion packet [$W\ m^{-1}\ K^{-1}$]
k_g	thermal conductivity of fluidizing gas [$W\ m^{-1}\ K^{-1}$]
k_s	thermal conductivity of solid particles [$W\ m^{-1}\ K^{-1}$]
M	mean of a distribution
N	number of packets involved in the heat transfer process
Nu_e	expected value of the Nusselt number, $\bar{h}_{p,e}d_p/k_g$
p	probability distribution function of packet contact period
p_i	instantaneous probability of the presence of i packets at a point on the surface
P	expected number of packets present at a point on the surface
Q_p	heat transferred from a packet to the surface [J]
Q_{pN}	heat transferred from N packets to the surface [J]
R_c	surface contact thermal resistance [$m^2\ K\ W^{-1}$]
R_p	packet thermal resistance [$m^2\ K\ W^{-1}$]
Re	Reynolds number, $Ud_p\rho_g/\mu$
Re_{mf}	Reynolds number at minimum fluidization, $U_{mf}d_p\rho_g/\mu$
t_c	contact period of a packet [s]
t_g	time between successive packets (interpacket period) [s]
t_t	total time between the arrival of successive packets [s]
T_B	bed temperature [K]
T_s	surface temperature [K]
U	superficial fluidizing velocity [$m\ s^{-1}$]
U_{mf}	superficial fluidizing velocity at minimum fluidization [$m\ s^{-1}$]
V	equilibrium distribution of interpacket periods
W	equilibrium distribution of packet contact periods
X	dimensionless variable, $R_p/R_c\sqrt{\pi}$
Y	square root of inverse packet time constant [$s^{-1/2}$]
Z	dimensionless variable defined by equation (29).

Greek symbols

α_g	thermal diffusivity of fluidizing gas [$m^2\ s^{-1}$]
ε	time-averaged bed voidage
ε_{mf}	time-averaged bed voidage at minimum fluidization
μ	viscosity of fluidizing gas [$kg\ m^{-1}\ s^{-1}$]
ρ_e	emulsion packet density [$kg\ m^{-3}$]
ρ_g	fluidizing gas density [$kg\ m^{-3}$]
ρ_s	density of solid particles [$kg\ m^{-3}$].

the vicinity of the contact point as well as conduction through the gas layer separating the particle and surface shows that very high heat transfer rates through the roughness elements occur only for the first 10–20 μs of the contact period. Subsequently, the contact Nusselt number remains practically unchanged. The value is also reasonably constant over a wide range of particle thermophysical properties and surface roughnesses. This probably explains why a simple constant like b is successfully able to represent the contact Nusselt number. The value of b ranges from 4 to 12 in the literature. From equations (1), (3) and (4), X can be expressed as

$$X = Yt^{1/2} = [tk_g^2b^2/k_eC_s\rho_ed_p^2]^{1/2}. \quad (5)$$

The amount of heat transferred per unit surface area during the packet contact period is therefore given by

$$\begin{aligned} Q_p &= \int_0^{t_c}(T_B-T_s)h_p(t)\,dt \\ &= [(T_B-T_s)/R_c]\int_0^{t_c}\exp(Y^2t)\,\mathrm{erfc}\,(Yt^{1/2})\,dt \\ &= [(T_B-T_s)/Y^2R_c][\mathrm{erf}\,(Yt_c^{1/2})\exp(Y^2t_c) \\ &\quad +(2Yt_c^{1/2}/\sqrt{\pi})-1]. \end{aligned} \quad (6)$$

Equations (6) highlights the influence of the packet contact period, t_c, on the packet heat transfer: the amount of heat transferred is zero for zero contact period, and increases with the square root of the contact period for very large values of t_c. Over a long period of time, the heat transferred will be the sum of the heat transfer due to the many packets that have contacted the surface. Thus, if N packets have contacted the surface, the total heat transfer is given by

$$Q_{pN} = [(T_B - T_s)/Y^2R_c] \sum_{i=1}^{N} [\mathrm{erfc}\,(Yt_{ci}^{1/2})\exp\,(Y^2t_{ci}) + 2(Yt_{ci}/\sqrt{\pi}) - 1] \quad (7)$$

where t_{ci} is the contact period of the ith packet. This heat transfer has occurred during the time period t_t where t_t is equal to

$$\left[\sum_{i=1}^{N} (t_{ci} + t_{gi})\right].$$

t_{gi} is the time period between the departure of the ith packet and the arrival of the $(i+1)$th packet and referred to here as the inter-packet period. The average particle convective heat transfer coefficient, $\bar{h}_p$, can then be defined as

$$\begin{aligned}\bar{h}_p &= Q_{pN}/[(T_B - T_s)t_t] \\ &= \sum_{i=1}^{N} [\mathrm{erf}\,(Yt_{ci}^{1/2})\exp\,(Y^2t_{ci}) + 2(Yt_{ci}/\sqrt{\pi}) - 1]/[Y^2R_c \sum_{i=1}^{N} (t_{gi} + t_{ci})]. \quad (8)\end{aligned}$$

In the case where the contact period of all packets is the same and the time periods between successive packets are equal, equation (8), without the summations, gives the value of $\bar{h}_p$ [6].

The contact periods and the time periods between packets, however, are stochastic variables and can take values between zero and infinity for a macroscopically undisturbed system. It is a simplification to assume $t_{ci} = (t_c)_{avg}$ and $t_{gi} = (t_g)_{avg}$. If this assumption is made, it implies that for a particular set of bed conditions, t_g and t_c have unique values, and measured values are random fluctuations about these unique values. In such a case, the experimentally obtained frequency distributions of t_c and t_g would be approximated by the normal curve. Experimental evidence, however, indicates that the t_c frequency distribution is best approximated by log-normal and gamma distributions [12]. Both these latter distributions are skewed and take values from zero to infinity. Additionally, the negative exponential distribution (which is a special case of the gamma distribution) is the only distribution having a truly Markovian character inasmuch as it is endowed with a complete lack of memory [13]. Consequently, the contact period of the ith packet is in no way affected by the contact periods of all the preceding $(i-1)$ packets. This experimental finding therefore argues for the treatment of the packet contact periods as a Markovian stochastic variable. This is carried out in the following section.

STOCHASTIC MODELLING OF PACKET CONTACT PERIODS

It is not possible to deterministically model the packet–surface contacting process. The packet departure is probably influenced by the pressure waves preceding bubbles (whose formation and distribution in the bed are themselves random), the local geometry and roughness of the surface, jetting of air from bubbles trapped upstream of the immersed surface, etc. The net effect of all these phenomena (seemingly random at the macroscopic level) is to impart a randomness to packet departure. Consequently, the packet contact period can be treated as a Markovian process. Specifically, it can be assumed that the probability of packet departure is constant throughout the contact period. For example, if the instantaneous probability of packet departure is 0.4 at the time of packet arrival, it is still 0.4 after the packet has been at the surface for 1 ms, or 10 ms, or 1 s: the length of the contact period does not influence the instantaneous probability of departure. This seems to be a fair assumption as the macroscopic bed characteristics remain unchanged during the contact period.

The probability distribution function (pdf) of the packet contact period is then given by the negative exponential distribution (see p. 458 of ref. [13]):

$$p(t) = C\exp\,(-Ct) \quad (9)$$

where C is the probability of packet departure at any instant (which is constant over the contact period). Equation (9) implies that the probability that a packet will have a contact period equal to t is given by $C\exp\,(-Ct)$.

Now, over a long period of time, a large number of packets contact the surface. These packets have different contact periods. Let $W(t)$ represent the contact period distribution of the packets. $W(t)$ is then given by (see p. 28 of ref. [14])

$$W(t) = F(t)/M \quad (10)$$

where $F(t)$ is the so-called 'survivor function', defined as

$$F(t) = \int_t^{\infty} p(x)\,\mathrm{d}x \quad (11)$$

and M is the expected value of the contact period

$$M = \int_0^{\infty} tp(t)\,\mathrm{d}t. \quad (12)$$

Thus

$$W(t) = \int_t^{\infty} p(x)\,\mathrm{d}x \Big/ \int_0^{\infty} tp(t)\,\mathrm{d}t. \quad (13)$$

Interestingly enough, the age distribution of packets over a large surface area at a given instant is also given by $W(t)$ and defined by equation (13) (see p. 61 of ref. [14]). This shows the statistical equivalence of considering the behaviour of packets at one point on the surface for a long period of time and that of packets over a large surface area at a given instant. This is a useful property that will be utilized later to incorporate time-averaged surface properties into equation (8), which has been developed for a local time-dependent process.

From equations (9) and (13)

$$W(t_c) = C\exp(-Ct_c). \tag{14}$$

Thus, for the particular case when the packet contact period pdf is exponential, the distribution of contact periods (or ages) is also exponential. The properties of the negative exponential distribution imply that the mean contact period would be $1/C$. This distribution is now used to develop the stochastic heat transfer model for particle convection.

HEAT TRANSFER MODEL

The average heat transfer coefficient obtained from the packet theory is given by equation (8). The summations in equation (8) are converted to integrals using the contact period pdf of equation (14) by the transformation

$$\sum_i f(t_{ci}) = \int_0^\infty W(t_c)f(t_c)\,\mathrm{d}t_c. \tag{15}$$

Thus

$$\bar{h}_p = (C/Y^2R_c)\left[\int_0^\infty [\operatorname{erf}(Yt_c^{1/2})\exp\{-(C-Y^2)t_c\} + (2Y/\sqrt{\pi})t_c\exp(-Ct_c) - \exp(-Ct_c)]\,\mathrm{d}t_c\right]\Bigg/\left[\int_0^\infty t_cW(t_c)\,\mathrm{d}t_c + \int_0^\infty t_gV(t_g)\,\mathrm{d}t_g\right] \tag{16}$$

where $V(t_g)$ is the distribution of inter-packet (or gas contact) periods. Solution of equation (16) yields

$$\bar{h}_p = \frac{[1/Y^2R_c][(Y^2/C)^{1/2}-1]/[1-(C/Y^2)]}{\int_0^\infty t_cW(t_c)\,\mathrm{d}t_c + \int_0^\infty t_gV(t_g)\,\mathrm{d}t_g}. \tag{17}$$

The first term in the denominator is equal to $1/C$, but evaluation of the second term is not possible until the form of $V(t_g)$ is known. At this point, it is assumed that the distribution of inter-packet periods at the surface is a Poisson process. Then $V(t_g)$ is also given by the exponential distribution (see p. 6 of ref. [15])

$$V(t_g) = D\exp(-Dt_g) \tag{18}$$

where $1/D$ is the mean inter-packet period. The second term in the denominator of equation (17) is then equal to $1/D$. The average heat transfer coefficient is therefore given by

$$\bar{h}_p = \frac{(1/R_c)[\{A^{1/2}-1\}/\{A-1\}]}{1+(C/D)} \tag{19}$$

where

$$A = Y^2/C. \tag{20}$$

Both C and D are system variables, and need to be determined experimentally. However, they are interrelated through the particle fraction at the surface. This interrelationship is discussed in the following.

The assumption of the inter-packet period being described by a Poisson process of rate D, and the packet contact period being described by an exponential distribution, equation (14), leads to the following expression for the probability that i packets are present at a point at a given instant (see p. 177 of ref. [15])

$$p_i = [(D/C)^i/i!]\Bigg/\left[\sum_{j=0}^{k}(D/C)^j/j!\right] \tag{21}$$

where k is the maximum number of packets that come into contact with the surface at any given instant. At a point on the surface, only one packet can come into contact with it at a given instant; hence k is one. The expected value of the number of packets at any point on the surface is therefore

$$\begin{aligned} P &= O\cdot p_0 + 1\cdot p_1 \\ &= [(0)(1)+(1)(D/C)]/[1+(D/C)] \\ &= (D/C)/[1+(D/C)]. \end{aligned} \tag{22}$$

Since P is the time-averaged expected number of packets at each point on the surface, and the maximum number of packets at each point can be 1, the ratio, $P/1$, is the time-fraction for which a point is covered by packets. Using the statistical equivalent of time-averaged and space-averaged properties brought out earlier (following equation (13))

$$P/1 = (D/C)/[1+(D/C)] = 1-\varepsilon \tag{23}$$

where ε is the time-averaged voidage. Thus

$$1+(C/D) = 1/(1-\varepsilon). \tag{24}$$

Equation (24) represents the theoretical development of an 'obvious' solution which is very widely used in fluidized bed heat transfer models. It can be visualized as the equality of the particle fraction at the surface with the fraction of time for which packets are present there.

Substituting in equation (19)

$$h_p = (1/R_c)(1-\varepsilon)[\{A^{1/2}-1\}/\{A-1\}]. \tag{25}$$

Variance of calculated $\bar{h}_p$

The value of h_p predicted by equation (19) is a mean or expected value: in practice, h_p values will range over a band. This variation occurs because of the uncertainty in the value of C which is calculated by regression of contact time data. This regression yields an expected value of C, $\bar{C}$, and its variance, Var (C).

The uncertainty in h_p can then be represented by its variance, Var $(\bar{h}_p)$, which is related to the variance of C as follows [16]:

$$\text{Var}\,(\bar{h}_p) = (\mathrm{d}\bar{h}_p/\mathrm{d}C)^2_{C=\bar{C}}\,\text{Var}\,(C). \tag{26}$$

Differentiating equation (19) with respect to C gives

$$(\mathrm{d}\bar{h}_p/\mathrm{d}C) = (\bar{h}_p/C)[\{(Y^2/C)/((Y^2/C)-1)\} - \{(Y^2/C)^{1/2}/(2((Y^2/C)^{1/2}-1))\} - \{(C/D)/(1+(C/D))\}]. \tag{27}$$

From equation (22)

$$(C/D)/\{1+(C/D)\} = \varepsilon. \tag{28}$$

From equations (26)–(28)

$$\text{Var}\,(\bar{h}_p) = (\bar{h}_{p,e}/\bar{C})^2\,\text{Var}\,(C)[\{\bar{A}/(\bar{A}-1)\} - \{\bar{A}^{1/2}/2(\bar{A}^{1/2}-1)\} - \varepsilon]^2 \tag{29}$$

where

$$\bar{h}_{p,e} = (1/R_e)(1-\varepsilon)[(\bar{A}^{1/2}-1)/(\bar{A}-1)]. \tag{30}$$

Thus

$$Nu_e = b(1-\varepsilon)[(\bar{A}^{1/2}-1)/(\bar{A}-1)] \tag{31}$$

and

$$\text{Var}\,(Nu) = (d_p/k_g)\,\text{Var}\,(\bar{h}_p). \tag{32}$$

COMPARISON WITH EXPERIMENTAL DATA

Evaluation of Nu_e using equation (31) requires the knowledge of three variables: b, ε and A. There is some dispute in the literature as to whether b should be 6 [10] or 12 [17]. The difference in the value of these two estimates is large, and comparison with experimental data should reveal which one provides a better fit. In other words, b is treated as an empirically determined constant. $(1-\varepsilon)$ is an easily measured quantity. In the absence of experimental data, ε can be evaluated from the following correlation for small particle beds [18]:

$$\varepsilon^3/(1-\varepsilon) = 4(Re/Ar)^{0.43}. \tag{33}$$

Grewal and Saxena [18] attribute an uncertainty of $\pm 10\%$ to equation (33) for beds with a wide variety of materials (glass beads, silicon carbide, nickel, copper, sand, coke, alumina, etc.), average particle diameters ranging between 167 and 1450 μm, atmospheric pressure, and temperature up to 1373 K. Packet properties required to compute Y^2 are calculated from equations (34) and (36) following the recommendations of Xavier and Davidson [10]:

$$(k_e/k_g) = (k_s/k_g)^Z + (0.1 d_p U_{mf}/\alpha_g) \tag{34}$$

where

$$Z = 0.28 - 0.757\log\varepsilon_{mf} - 0.057\log(k_s/k_g) \tag{35}$$

and

$$\rho_e = \rho_s(1-\varepsilon_{mf}). \tag{36}$$

Table 1. Properties of the 245 μm glass beads [12]

Particle thermal conductivity	0.89 W m^{-1} K^{-1}
Particle heat capacity	753.6 J kg^{-1} K^{-1}
Particle density	2470 kg m^{-3}
Slumped bed void fraction	0.39
Minimum fluidizing velocity	0.0588 m s^{-1}

It is implicit in equations (34)–(36) that the packet consists of incipient fluidized emulsion.

In this comparison, the values of C have been taken from the data of Ozkaynak and Chen [12] for the contact period distributions in a bed of 245 μm glass beads at three different velocities. This particular data set has been chosen as simultaneous measurements of the packet contact period distribution and the heat transfer coefficient are available. The frequency data, graphically presented by Ozkaynak and Chen [12], are fitted to the form of equation (14) and the maximum likelihood value of C, $\bar{C}$, is obtained. Table 1 presents the properties of the 245 μm glass beads. Figure 1 shows a comparison of the best-fit Poisson distributions with the experimental data. It should be noted that owing to the nature of the Poisson distribution, the best-fit was estimated with the contact period data in deciseconds (rather than seconds). Consequently, the constant within the exponential term in equation (14) becomes ten times the best-fit value of $\bar{C}$ given in Table 2 when the contact period is in seconds.

Table 2 lists the regressed values of $\bar{C}$ (and the associated variance) for the three cases. The value of Y^2 for the bed (which is not affected by velocity) is $0.0674b^2$. The bed is assumed to be at 300 K and 1 atm. The computed values of $\bar{h}_{p,e}$ and Var $(\bar{h}_p)$ at the three velocities are also listed in Table 2, as are the experimental values of h_p at these conditions as measured by Ozkaynak and Chen [12]. A comparison of the calculated $\bar{h}_{p,e}$ values with the experimental data of Ozkaynak and Chen [12] over the complete velocity range is shown in Fig. 2 for the two values of b. The error bar on the calculated values indicates one standard deviation (square root of the variance of h_p) on either side of $\bar{h}_{p,e}$.

Excellent agreement of the model predictions with a b value of 12 occurs with the experimental data. The expected values slightly overpredict the experimental values, but this overprediction is always less than 6% and well within the one standard deviation range shown in Fig. 2.

CONCLUSIONS

A stochastic picture of the particle convection process is drawn and used to develop a heat transfer model. Unfortunately, due to the paucity of simultaneous particle contact period distribution data and heat transfer data, model predictions could be compared with experimental data at three points only. At these three points, excellent agreement is obtained

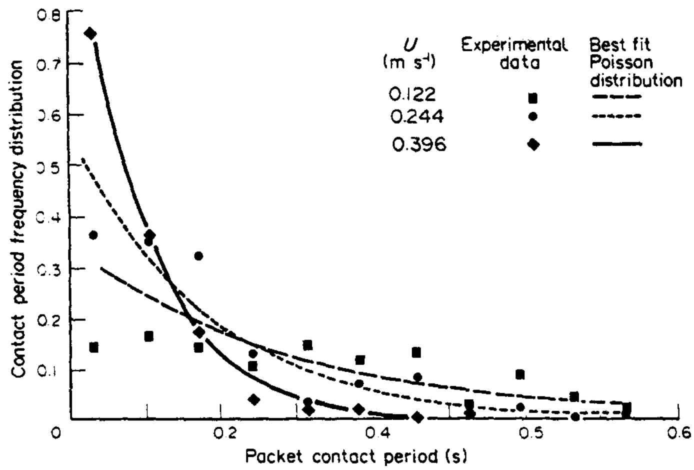

FIG. 1. The equilibrium contact period data of Ozkaynak and Chen [12] for three velocities in a bed of 245 μm glass beads and the corresponding best-fit Poisson distributions.

Table 2. Computed properties of the beds of 245 μm glass beads at different velocities

	U (m s^{-1}) 0.122	0.244	0.396
C (s^{-1})	0.3392	0.5781	1.087
Var (C) (s^{-2})	0.1046	0.3039	1.4768
ε	0.4985	0.5365	0.5635
$\bar{h}_{p,c}$ (W m^{-2} K^{-1})			
$b = 6$	282.07	268.9	260.0
$b = 12$	565.5	537.9	520.0
Var ($\bar{h}_p$)			
$b = 6$	1386.55	1570.10	2352.68
$b = 12$	5572.96	6282.75	9410.73
$\sigma(\bar{h}_p)$			
$b = 6$	37.2	39.6	48.5
$b = 12$	74.7	79.26	97.0
$\bar{h}_{p,exp}$	543.9	521.9	492.0

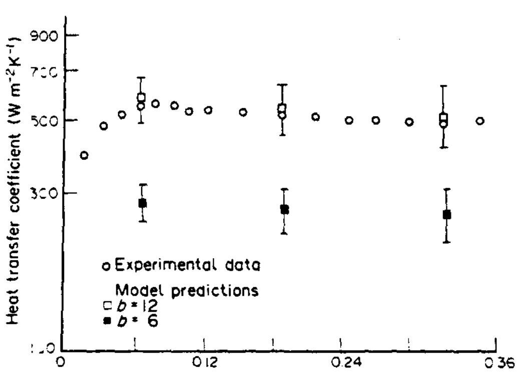

FIG. 2. Comparison of heat transfer coefficient predicted by the model for two values of b with the experimental data of Ozkaynak and Chen [12]. The vertical bars on the predicted values represent one standard deviation on either side.

between the expected value of the particle convective heat transfer coefficient (predicted by the model) and the experimentally measured overall heat transfer coefficient. The overall heat transfer coefficient can be considered equivalent to the particle convective heat transfer coefficient here since this data pertains to a bed of small particles (245 μm glass beads) at ambient conditions in which the gas convective and radiative components of heat transfer are negligible.

This limited comparison also clearly shows that the contact Nusselt number is 12 (as previously shown by Gloski *et al.* [17]). In a sense, this confirms the non-smooth particle surface contact picture of Decker and Glicksman [11], and brings out the fact that the majority of heat transfer occurs through the microscopic-level gas gaps between the surface and the particle. The model development also brings out the assumptions which underlie the equivalence of the average heat transfer coefficient and the product of the local heat transfer coefficient and the particle concentration: an exponential distribution of particle contact periods, and a constant-rate Poisson distribution of interpacket periods.

Much more comprehensive testing of the model is required over a range of particle sizes, bed pressures and temperatures. The velocity dependence is shown to be well represented by the model.

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UN MODELE STOCHASTIQUE POUR LA CONVECTION THERMIQUE SUR UNE PARTICULE DANS DES LITS FLUIDISES GAZ–SOLIDE

Résumé—La convection de particule est le mode prédominant de transfert thermique dans des lits fluidisés de petites particules à des températures inférieures à 650 K et à des pressions proches de l'ambiance. Le mécanisme de transfert de chaleur est la conduction variable entre la surface et les particules du lit chaud qui sont en contact avec elle d'une façon récurrente et apériodique. Le contact particule–surface est traité comme un processus stochastique avec des périodes qui sont décrites par une distribution de Poisson et les périodes de contact par une distribution exponentielle négative. Ces mécanismes de contact par une distribution exponentielle négative. Ces mécanismes de contact sont incorporés dans le modèle de transfert de Baskakov et des expressions sont obtenues pour le coefficient de transfert thermique et sa variance. Les données de distribution de période de contact de Ozkaynak et Chen pour un lit de billes de verre de 245 μm à trois vitesses de fluidisation sont employées pour prédire le coefficient de transfert thermique et sa variance aux trois vitesses. Les coefficients ainsi calculés sont trouvés être en excellent accord avec les coefficients mesurés. Néanmoins, la rareté des données simultanées sur la distribution des périodes de contact et des transferts de chaleur limite la vérification de ce modèle.

EIN STOCHASTISCHES MODELL FÜR DIE WÄRMEÜBERTRAGUNG DURCH PARTIKELKONVEKTION IN WIRBELSCHICHTEN

Zusammenfassung—Partikelkonvektion ist der dominierende Mechanismus der Wärmeübertragung in Gas-Festkörper-Wirbelschichten kleiner Partikelgröße, Temperaturen unterhalb 650 K und nahezu Umgebungsdruck. Der Wärmeübergang erfolgt durch instationäre Leitung zwischen der Wärmetauscheroberfläche und den heißen Partikeln, die sich in einer unregelmäßig wiederkehrenden Weise berühren. Die Berührung der Partikel wird als stochastischer Vorgang behandelt, der zwischen einer Kontaktphase mit der Wärmetauscherfläche und einer Phase, in der sich die Partikel in der Schicht befinden, unterscheidet. Die erste Phase wird durch eine Poisson-Verteilung beschrieben, die zweite durch eine negative Exponentialverteilung. Diese Merkmale des Kontaktvorgangs werden in das Modell von Baskakov eingeführt; es ergeben sich Austrücke für den Wärmeübergangskoeffizienten und seine Schwankung. Die Verteilungsdaten von Ozkaynak und Chen für die Kontaktperiode in einem Wirbelbett mit Glaskugeln (Durchmesser 245 μm) bei drei unterschiedlichen Fluidisierungsgeschwindigkeiten werden zur Berechnung von Wärmeübergangskoeffizienten und deren Schwankung verwendet. Die Übereinstimmung mit Meßwerten ist hervorragend. Die Verifikation des Modells wird jedoch durch einen Mangel an Daten für die Verteilung der Kontaktperiode und den Wärmeübergang eingeschränkt.

СТОХАСТИЧЕСКАЯ МОДЕЛЬ КОНВЕКТИВНОГО ПЕРЕНОСА ТЕПЛА ЧАСТИЦАМИ В ПСЕВДООЖИЖЕННЫХ СЛОЯХ

Аннотация—Конвекция частиц является доминирующим фактором теплопереноса в псевдоожиженных слоях мелких частиц при температурах ниже 650 К и давлениях, близких к давлению окружающей среды. Перенос тепла осуществляется путем нестационарной теплопередачи между поверхностью теплообмена и горячими частицами, находящимися в возвратно-апериодическом контакте с ней. Контакт между частицами и поверхностью рассматривается как стохастический процесс, межпакетные периоды которого описываются распределением Пуассона, а периоды контакта—отрицательным экспоненциальным распределением. Эти особенности процесса контакта введены в предложенную Баскаковым модель пакетного теплопереноса, и получены выражения для расчета коэффициента теплопереноса и его вариаций. Данные Озкайнака и Чена по распределению периодов контакта для слоя стеклянных шариков размером 245 мкм при трех скоростях псевдоожижения используются для расчета коэффициента теплопереноса и его вариаций при этих скоростях. Найдено, что расчетные коэффициенты теплопереноса хорошо согласуются с полученными экспериментально. Однако малое число одновременно полученных данных по распределениям периодов контакта и теплопереносу ограничивают возможность проверки адекватности данной модели.